

Indicial Equations

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Introduction

An indicial equation is one in which the power is the unknown, e.g. $2^n = 8$, the solution of which would be $n = 3$ because $2^3 = 8$.

Indicial equations often occur in the calculation of compound interest.

How long should an investment be left before it doubles in value?

Consider the case of a savings account paying 5% interest, compounded annually, where an initial deposit of £ 500 is made.

After **1** year the capital, £ 500, will have increased to $£ 500 \times 1.05 = £ 500 \times (1.05)^1$

After **2** years to $£ 500 \times 1.05 \times 1.05 = £ 500 \times (1.05)^2$

After **3** years to $£ 500 \times (1.05) \times (1.05) \times (1.05)$
 $= £ 500 \times (1.05)^3$

Continuing in this way we can see that after a number of years, n say, the original capital of £ 500 will have grown to $£ 500 \times (1.05)^n$.

The question of how long should the investment be left in order for the capital to double would then require us to solve the equation $£ 500 \times (1.05)^n = £ 1000$

After dividing both sides of the equation by 500 this equation becomes $(1.05)^n = 2$, an indicial equation.

Method of Solution

To solve the equation $(1.05)^n = 2$ requires a knowledge of the properties of logarithms, in particular the law of logarithms which states that:

$$\log[m^n] = n \log[m]$$

Taking the logarithm of each side of the equation $(1.05)^n = 2$ will give us the equation

$$\log[(1.05)^n] = \log[2]$$

which after applying the above law of logarithms simplifies to

$$n \log[1.05] = \log[2]$$

Re-arranging this equation gives:

$$n = \frac{\log[2]}{\log[1.05]} = 14.21 \quad (2\text{dp})$$

If interest is added annually the original investment will therefore take 15 complete years to at least double.

General Solution

Applying the previous method to the solution of the equation $a^n = b$, when we know the values of a and b , leads to the following general solution.

$$n = \frac{\log b}{\log a}$$

Exercises

In each of the following questions determine the number of complete years needed to achieve the target. All of the questions assume that interest is compounded annually.

1. Initial investment £ 5000 invested at 3% per annum. Target £ 10000
2. Initial investment £ 5000 invested at 5% per annum. Target £ 12000
3. Initial investment £ 1000 invested at 4.2% per annum. Target £ 8000
4. Initial investment £ 2500 invested at 3.2% per annum. Target £ 4000
5. Initial investment £ 100 invested at 10% per annum. Target £ 1000

Answers

1. 24 years
2. 18 years
3. 51 years
4. 15 years
5. 25 years