

Force as a vector

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As described in leaflet 1.1. (Introduction to Mechanics) vector quantities are quantities that possess both magnitude and direction. A force has both magnitude and direction, therefore:

Force is a vector quantity; its units are newtons, N.

Forces can cause motion; alternatively forces can act to keep (an) object(s) at rest. Further information on types of force can be found in leaflet 2.3 (Types of Force).

A **resultant force** is the single force which represents the vector sum of two or more forces. For example, two forces of magnitudes F_1 , F_2 acting upon a particle have a resultant as shown:



In order to add two forces together, the 'start' of the second force needs to be moved to the 'end' of the first force, with the resultant going from the start of the first force directly to the end of the second force (as shown in the diagram). This rule is then extended when considering more forces.

Worked Example 1.

Consider two forces of magnitudes 5 N and 7 N acting on a particle, with an angle of 90° between them. What is the magnitude and direction of the resultant force?



Solution

Firstly the forces need to be placed in the configuration described earlier to form a triangle as seen in the right-hand diagram above. In this instance the resultant can be calculated using Pythagoras' theorem.

In addition, using standard trigonometry, the angle α can be found.

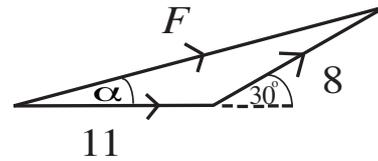
$$\begin{aligned} F^2 &= 7^2 + 5^2 = 74 \\ \Rightarrow F &= \sqrt{74} = 8.6 \text{ N (2 s.f.)} \end{aligned}$$

$$\begin{aligned} \tan \alpha &= \frac{5}{7} \\ \Rightarrow \alpha &= 36^\circ \text{ (2 s.f.)} \end{aligned}$$

Hence the resultant has magnitude 8.6 N at 36° above the positive x-axis.

Worked Example 2.

Consider two forces of magnitudes 11 N and 8 N acting on a particle, with an angle of 30° between them. What is the magnitude and direction of the resultant force?



Solution. In order to calculate the required magnitude it is necessary to use the cosine rule. Pythagoras' theorem cannot be used as the forces are not perpendicular.

$$F^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \cos 150^\circ = 337.42$$

$$F = \sqrt{337.42} = 18.37 = 18 \text{ N (2 s.f.)}$$

In order to calculate the required angle, α , which F makes with the positive x-axis, the sine rule is needed.

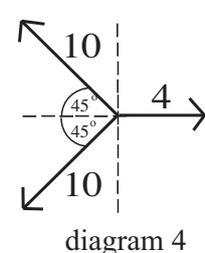
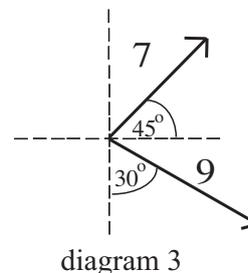
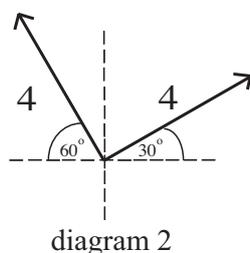
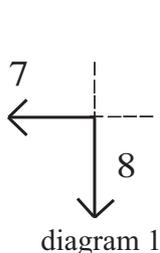
$$\frac{\sin \alpha}{8} = \frac{\sin 150^\circ}{F} \quad \therefore \sin \alpha = \frac{8}{18.37} \sin 150^\circ \quad \Rightarrow \alpha = 13^\circ \text{ (2 s.f.)}$$

Therefore, the resultant force, F , of the two forces has a magnitude of 18 N at an angle of 13° to the positive x-axis.

The same principles can be applied to a system of more than 2 forces. In practice this can sometimes be difficult. In these cases it is often easier to sum the components of the forces involved.

Exercises

1. Consider two forces of magnitudes 7 N and 8 N acting on a particle, as shown in diagram 1. What is the magnitude and direction of the resultant force?
2. Consider two forces of magnitudes 4 N and 4 N acting on a particle, as shown in diagram 2. What is the magnitude and direction of the resultant force?
3. Consider two forces of magnitudes 7 N and 9 N acting on a particle, as shown in diagram 3. What is the magnitude and direction of the resultant force?
4. For the set of forces in diagram 4, what is the magnitude and direction of the resultant?



Answers (all 2 s.f.)

1. $F = 11 \text{ N}$ at 49° below the negative x-axis
2. $F = 5.7 \text{ N}$ at 75° above the positive x-axis
3. $F = 9.9 \text{ N}$ at 17° below the positive x-axis
4. $F = 10 \text{ N}$ in direction of negative x-axis.