Resolving Forces - i, j notation

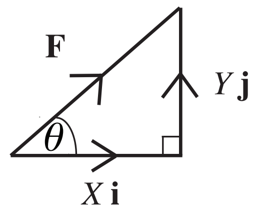
mc-web-mech1-7-2009

The method to find a resultant, as used in leaflet 1.5 (Force as a Vector), is generally slow and can be complicated. Taking components of forces can be used to find the resultant force more quickly. In two dimensions, a force can be resolved into two mutually perpendicular components whose vector sum is equal to the given force. The components are often taken to be parallel to the x- and y-axes. In two dimensions we use the perpendicular unit vectors **i** and **j** (and in three dimensions they are **i**, **j** and **k**).

Let **F** be a force, of magnitude F with components X and Y in the directions of the x- and y-axes, respectively.

Then, $\mathbf{F} = X\mathbf{i} + Y\mathbf{j}$
and from the right-angled triangle,
$X = F\cos\theta$, and $Y = F\sin\theta$

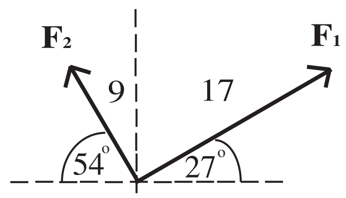
$$\therefore \mathbf{F} = F\cos\theta\mathbf{i} + F\sin\theta\mathbf{j}.$$



Also F $= \sqrt{X^2 + Y^2}$ and $\tan\theta = \dfrac{Y}{X}$

Worked Example 1.

Consider two forces, $\mathbf{F_1}$ and $\mathbf{F_2}$, of magnitudes 17 N and 9 N acting on a particle, as shown in the diagram. What is the magnitude and direction of the resultant force, $\mathbf{F}$?



Solution

The method employed is to resolve $\mathbf{F_1}$ and $\mathbf{F_2}$ into horizontal and vertical components and then add these to obtain the components of the resultant.

$$\begin{aligned}\mathbf{F_1} &= (17\cos 27°\mathbf{i} + 17\sin 27°\mathbf{j})\\ \mathbf{F_2} &= (-9\cos 54°\mathbf{i} + 9\sin 54°\mathbf{j})\end{aligned}$$

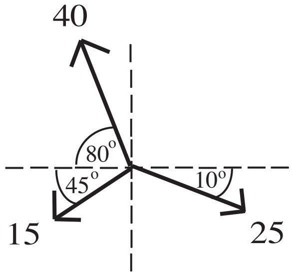
$$\begin{aligned}\text{Resultant,}\quad \mathbf{F} &= \mathbf{F_1} + \mathbf{F_2}\\ &= (17\cos 27°\mathbf{i} + 17\sin 27°\mathbf{j}) + (-9\cos 54°\mathbf{i} + 9\sin 54°\mathbf{j})\\ &= (17\cos 27° - 9\cos 54°)\mathbf{i} + (17\sin 27° + 9\sin 54°)\mathbf{j}\\ &= 9.857\mathbf{i} + 14.999\mathbf{j}\end{aligned}$$

$$\begin{aligned}\text{Magnitude, F} &= \sqrt{9.857^2 + 14.999^2} = 17.948 = 18 \text{ N (2 s.f.)}\\ \text{Angle, }\theta &= \tan^{-1}\frac{14.999}{9.857} = 56.688° = 57° \text{ (2 s.f.) above the positive x-axis.}\end{aligned}$$

Written by T. Graham, M.C. Harrison, S. Lee, C.L.Robinson

Worked Example 2.

For the set of forces shown in the diagram, what is the magnitude of the resultant force and at what angle does it act?



Solution

When there are several forces involved it is sometimes easier to immediately collect all **i** components then all **j** components, i.e. resolve horizontally then vertically.

Resolving horizontally: $25\cos 10^\circ - 40\cos 80^\circ - 15\cos 45^\circ = 7.068$

Resolving vertically: $40\sin 80^\circ - 15\sin 45^\circ - 25\sin 10^\circ = 24.445$

Therefore the resultant has magnitude $\sqrt{7.068^2 + 24.445^2} = 25.45 = 25$ (2 s.f.)
acting at an angle of $\tan^{-1}\dfrac{24.445}{7.068} = 73.87^\circ = 74^\circ$ (2 s.f.) above the positive x-axis.

Notes

1. It is not always necessary to resolve in the directions of **i** and **j**, see leaflets 2.6/ 2.7 (Forces at an Angle/ Motion on a Slope).
2. The method is also applicable to work in three dimensions, as each force can be resolved into **i**, **j** and **k** components.

Exercises

1. Consider two forces of magnitudes 20 N and 16 N acting on a particle, as shown in diagram 1. What is the magnitude and direction of the resultant force?
2. Consider two forces of magnitudes 26 N and 28 N acting on a particle, as shown in diagram 2. What is the magnitude and direction of the resultant force?
3. For the set of forces shown in diagram 3, what is the magnitude and direction of the resultant force?
4. For the set of forces shown in diagram 4, what is the magnitude and direction of the resultant force?

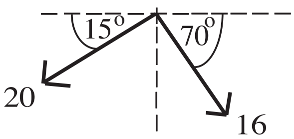


diagram 1

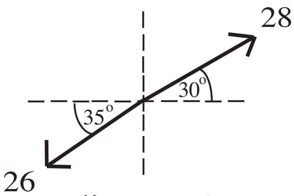


diagram 2

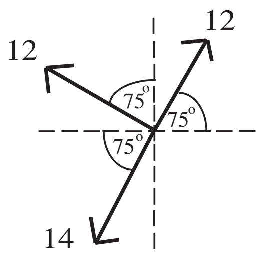


diagram 3

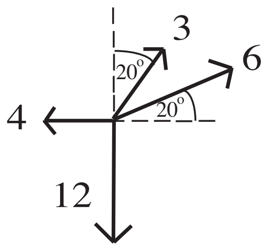


diagram 4

Answers (all 2 s.f.)
1. F = 24 N at 56° below the negative x-axis
2. F = 3.1 N at 17° below the positive x-axis
3. F = 12 N at 5.5° above negative x-axis.
4. F = 7.6 N at 70° below the positive x-axis.

Written by T. Graham, M.C. Harrison, S. Lee, C.L.Robinson