Forward prices

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A forward contract is an agreement between two parties to buy or sell an asset at a specified future time at a price agreed today. The forward price is the agreed price of an asset in a forward contract. The price is paid at maturity — that is, the time at which the asset changes hands — and there is no payment by either party when the contract is first entered into.

Throughout this leaflet we consider a forward contract agreed at time 0 ("now") to purchase an asset at a future time T. For simplicity we assume that holding the asset neither provides income nor incurs costs in the period $[0, T]$.

The forward price formula

The forward price F is given by

$$F = S_0 e^{rT}$$

where S_0 is the spot price of the asset at time 0 and r is the risk-free interest rate, which we assume is constant.

Justification of the forward price formula

This is an illustration of the "no arbitrage" principle"; that is, market prices are such that no risk-free profits are available from buying and selling in that market. Suppose $F > S_0 e^{rT}$ then investors adopting the following strategy

1. At time 0 borrow S_0 and buy one unit of the asset on the spot market
2. At time 0 enter into a contract to supply one unit of the asset at time T for price F (this is known as "shorting the forward contract")
3. At time T sell the asset for F as agreed
4. At time T use F to pay off the loan, which has now increased to $S_0 e^{rT}$

would make a risk-free profit of $F - S_0 e^{rT}$, contrary to the no-arbitrage principle.

The time value of a forward contract

The forward price is set so that a forward contract has zero value at time 0. However, changes to the spot price of the asset cause the contract to have a non-zero value at time $t \in [0, T]$ during its lifetime, namely

$$F - S_t e^{r(T-t)}$$

where S_t is the spot price of the asset at time t.

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Liverpool John Moores University

Reviewer: Vassili Kolokoltsov
Warwick University

Example

The current spot price of an asset is $£228$ and the interest rate is $r = 6.75\%$ per annum. Find the one-month and six-month forward prices for the asset.

Solution

For the one-forward, the forward price formula gives $£228e^{0.0675/12} = £229.29$ and for the six-month forward $£228e^{6\times 0.0675/12} = £235.83$

Example (continued)

Suppose a two-month forward contract on the asset is trading with forward price $F_2 = £231.18$. Show that the following strategy:

- Short the two-month forward
- Borrow $£228$
- Buy the asset

is an arbitrage strategy with zero initial cost realising a positive gain in two month's time.

Solution

Borrowing $£228$ enables the asset to be bought and the forward costs nothing initially. After two months, disposing of the asset via the forward contract realises $£231.18$, repaying the loan costs $£228e^{0.0675/6} = £230.58$ giving a profit of $£0.60$.

Exercise

What is the correct three-month forward price in the above example?

Answer

The correct forward price is $£228e^{0.0675/4} = £231.88$.

Exercise

Reverse the strategy given overleaf to show that if $F < S_0e^{rT}$ then investors can realise a risk-free profit. *Hint: it is necessary that the asset can be short sold — that is, it is possible to borrow the asset from a third party and sell it on the spot market with the intention of buying the identical asset at a later date to return to the lender.*

Suppose a three-month forward contract on the asset discussed above is trading with forward price $£231.48$; what is the risk-free profit which can be made?

Answer

1. At time 0 short sell one unit of the asset on the spot market and invest S_0
2. At time 0 enter into a contract to buy one unit of the asset at time T for price F
3. At time T recover the loan, which has now increased to S_0e^{rT}
4. At time T buy the asset for F as agreed and return the borrowed asset

would make a risk-free profit of $S_0e^{rT} - F$, contrary to the no-arbitrage principle.

For the figures quoted, this risk-free profit is $£228e^{0.0675/4} - £231.48 = £0.40$.

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Warwick University