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Symmetric matrices and the transpose of a matrix

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This leaflet will explain what is meant by a **symmetric matrix** and the **transpose** of a matrix.

Symmetric matrices

A **symmetric** matrix is a square matrix which is symmetric about its leading diagonal (top left to bottom right). The following are symmetric matrices:

$$M = \begin{pmatrix} 4 & -1 \\ -1 & 9 \end{pmatrix} \qquad N = \begin{pmatrix} 2 & 7 & 3 \\ 7 & 9 & 4 \\ 3 & 4 & 7 \end{pmatrix}$$

Note that the leading diagonal is a line of symmetry - a mirror line.

The transpose of a matrix

If the rows and columns of a matrix A are interchanged (so that the first row becomes the first column, the second row becomes the second column, and so on) we obtain what is called the **transpose** of A, denoted A^T. For example, if $A = \begin{pmatrix} 4 & -1 \\ 13 & 9 \end{pmatrix}$, then by interchanging rows and columns, we obtain $A^T = \begin{pmatrix} 4 & 13 \\ -1 & 9 \end{pmatrix}$.

The transpose of a symmetric matrix

Consider again matrices M and N above.

$$M = \begin{pmatrix} 4 & -1 \\ -1 & 9 \end{pmatrix} \qquad N = \begin{pmatrix} 2 & 7 & 3 \\ 7 & 9 & 4 \\ 3 & 4 & 7 \end{pmatrix}$$

Taking the transpose of each of these produces

$$M^T = \begin{pmatrix} 4 & -1 \\ -1 & 9 \end{pmatrix} \qquad N^T = \begin{pmatrix} 2 & 7 & 3 \\ 7 & 9 & 4 \\ 3 & 4 & 7 \end{pmatrix}$$

Observe that when a matrix is symmetric, as in these cases, the matrix is equal to its transpose, that is,

$$M = M^T \qquad \text{and} \qquad N = N^T.$$

If A is any symmetric matrix, then $A = A^T$

A further example of a transpose

Here is another example:

If $C = \begin{pmatrix} 7 & 1 \\ -3 & 2 \\ 4 & 4 \end{pmatrix}$ then $C^T = \begin{pmatrix} 7 & -3 & 4 \\ 1 & 2 & 4 \end{pmatrix}$.

Note that whereas C is a 3×2 matrix, its transpose, C^T, is a 2×3 matrix. More generally, if C is an $m \times n$ matrix, its transpose, C^T, is a $n \times m$ matrix.

The next leaflets in the series will show the conditions under which we can add, subtract and multiply matrices.

Note that a video tutorial covering the content of this leaflet is available from **sigma**.