mcccp-richard-5

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Differentiation for Economics and Business Studies
Functions of one variable

This leaflet is an overview of differentiation and its applications in Economics.

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The **derivative** of a function f is a new function obtained by **differentiating** f. It can be written f' or $\frac{df}{dx}$. It is the **rate of change** of f and gives information on the shape and optimum values of f.

Table of Derivatives

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
k constant	0
x	1
x^2	$2x$
x^n	nx^{n-1}
e^x	e^x
$\ln x$	$\frac{1}{x}$
e^{ax+b}	ae^{ax+b}
$\ln(ax+b)$	$\frac{a}{ax+b}$
$\ln(f(x))$	$\frac{f'(x)}{f(x)}$

Rules of Differentiation

For any function f and g and any constant value k:

Additive constant: if $y = f(x) + k$ then $\frac{dy}{dx} = f'(x)$

Multiplicative constant: if $y = kf(x)$ then $\frac{dy}{dx} = kf'(x)$

Addition rule: if $y = f(x) \pm g(x)$ then $\frac{dy}{dx} = f'(x) \pm g'(x)$

Product rule: if $y = f(x) \times g(x)$ then

$$\frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$$

Quotient rule: if $y = \frac{f(x)}{g(x)}$ then

$$\frac{dy}{dx} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Chain rule (derivative of a function of a function):

if $y = g(u)$ with $u = f(x)$ then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = g'(u) \times f'(x)$$

Shape of Function

sign of $\frac{dy}{dx} = f'(x)$	sign of $\frac{d^2y}{dx^2} = f''(x)$	shape of the curve of f
> 0	> 0	increasing and convex
> 0	< 0	increasing and concave
< 0	> 0	decreasing and convex
< 0	< 0	decreasing and concave

Stationary points

First Order Condition (FOC): if a point x_0 is such that $f'(x_0) = 0$, then it is a stationary point. It can be a maximum, or a minimum, or an inflection point.

Second Order Condition (SOC): the sign of the second derivative indicates whether the optimum is a maximum, minimum or inflection point:

value of $\frac{dy}{dx}(x_0) = f'(x_0)$	sign of $\frac{d^2y}{dx^2}(x_0) = f''(x_0)$	Nature of point at x_0
0	> 0	minimum
0	< 0	maximum
0	0	inflection

Derivatives in Economics

The **marginal cost** MC is the rate of change of the total cost function TC: $MC = \frac{dTC}{dq}$, where q is the output. Similarly, the **marginal revenue** MR is the rate of change of the total revenue function TR: $MR = \frac{dTR}{dq}$. When MR is positive, TR is an increasing function of q, and when MR is negative, TR is a decreasing function of q.

The **elasticity** E of a function $q = f(p)$ is the rate of proportionate change in q given a proportionate change in p: $E = \frac{\frac{dq}{q}}{\frac{dp}{p}} = \frac{d\ln q}{d\ln p}$. This is the slope of the function when plotted on a log-log scale.

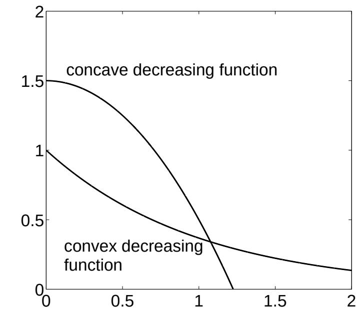

Figure 1: Examples of decreasing concave and convex functions

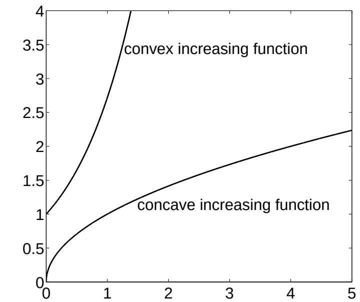

Figure 2: Examples of increasing concave and convex functions